University College Dublin
An Coláiste Ollscoile, Baile Átha Cliath

SEMESTER 1 MOCK EXAMINATION 2017/2018

MATH00030

Access to Science, Engineering and Agriculture: Mathematics 1

Dr. Anthony Brown*

Time Allowed: 2 hours

Instructions and Notes for Candidates

Candidates should attempt all questions.
Not all questions are allocated the same number of marks.
The exam is marked out of 100 marks.

Notes for Invigilators

Non programmable calculators are permitted.
The formula sheet provided is permitted.

1. (a) Without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\dfrac{3}{7} - \dfrac{4}{9}$

(ii) $-\dfrac{2}{7} \times \left(-\dfrac{5}{7}\right)$

(iii) $\dfrac{2}{9} \div \dfrac{11}{5}$

(iv) -6^2

(v) $\left(\dfrac{16}{81}\right)^{-\frac{3}{4}}$

(vi) $6 \div (7 - (-9) \times (-8))$

(vii) $\log_4 64$

(viii) $\log_3 \dfrac{1}{27}$ [8]

(b) Simplify the following expressions by expressing them as a single power of x.

(i) $x^6 \times x^{-8}$

(ii) $x^{\frac{1}{2}} \div x^{-\frac{2}{3}}$

(iii) $(x^{-2})^{-3}$ [3]

(c) Express $\log_a \left(\left(\dfrac{y^3}{x^4}\right)^{-2}\right)$ in terms of $\log_a x$ and $\log_a y$ [2]

(d) (i) Approximate 9.94999 to one decimal place.

(ii) Approximate 0.0004454 to two significant figures.

(iii) Express 132410.01 in scientific notation.

(iv) Express 0.000249 in scientific notation to one significant figure. [4]

(e) Simplify $(3x^2 - 2x + 3) - (-3x - 3)$. [1]

(f) Multiply out $(2x^4 - 3x^2)(-3x^2 + 4)$. [2]

(g) Perform long division on $\dfrac{x^2 + 5x + 2}{x + 3}$, giving the quotient and remainder. [4]

(h) Evaluate $\displaystyle\sum_{i=-3}^{2} -i^3$ [2]

(i) Calculate $\dbinom{7}{3}$ without using a calculator.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator. [2]

(j) Expand $(2x - 3y)^3$ using The Binomial Theorem. [4]

2. (a) Find the equation of the line through the point $(-1, 1)$ parallel to the line $y = 3x - 1$. **[2]**

(b) Solve the simultaneous equations

$$-3x + 4y = 11$$
$$2x - 3y = -8$$

[3]

(c) Find the midpoint of the line segment joining $(-1, -2)$ and $(2, 3)$ **[1]**

3. (a) Write the expression $2x^2 - 3x + 1$ in completed square form. **[3]**

(b) Solve the equation $2x^2 - 3x + 1 = 0$ using the completed square form you found in Part(a). **[2]**

(c) Sketch the graph of the function $y = 2x^2 - 3x + 1$, showing the y-intercept, the x-intercept(s) (if applicable) and the turning point. **[4]**

4. (a) For each of the following:

- Say whether or not it is a function and if not say why not.
- If it is a function state the domain and the codomain.

(i)

$$f \colon \mathbb{R}^- \to \mathbb{R}^-$$
$$x \mapsto -2x - 1$$

(ii)

$$f \colon \mathbb{R}^- \to \mathbb{R}^+$$
$$x \mapsto x^2 + 1$$

[4]

(b) Sketch the graph of the function

$$f \colon \{-4, -2, 0, 1, 3\} \to \{-3, -2, 0, 2, 3\}$$
$$-4 \mapsto 2$$
$$-2 \mapsto -2$$
$$0 \mapsto 2$$
$$1 \mapsto 0$$
$$3 \mapsto 3$$

[2]

(c) For each of the following functions, say whether they are injective, surjective or bijective. If a function is not injective or surjective then say why not.

(i)

$$f\colon \{1,2,3,4\} \to \{A,B,C,D\}$$
$$1 \mapsto B$$
$$2 \mapsto A$$
$$3 \mapsto D$$
$$4 \mapsto B$$

(ii)

$$f\colon \mathbb{R}^- \to \mathbb{R}^-$$
$$x \mapsto 2x - 1$$

[3]

(d) State whether each of the functions in Part (c) has an inverse function or not, giving a reason in each case. [2]

(e) Solve the equation $8^{3x} = 7$. [3]

5. (a) Convert $105°$ to radians, leaving your answer as a multiple of π. [1]

(b) Convert $\dfrac{7\pi}{4}$ radians to degrees. [1]

(c) Using the geometric method, find $\tan\left(-\dfrac{2\pi}{3}\right)$ without using a calculator. [3]

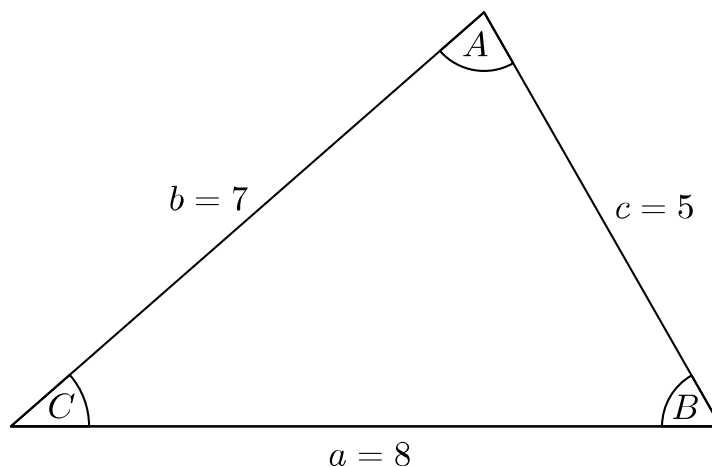(d) Find the size of the angle B in the triangle in Figure 1. [3]

Figure 1: The triangle for Question 5 (d).

(e) Using whichever trigonometric formulae you like, but without using a calculator, calculate the following.

Note that you should show enough of your working to demonstrate that you have not simply entered the expression into a calculator.

(i) $\sin\left(\dfrac{5\pi}{4}\right)$

(ii) $\tan\left(-\dfrac{\pi}{12}\right)$ [4]

6. (a) Find the derivative of $f(x) = 2x^2$ using first principles. [2]

(b) Find the derivatives of the following functions.

(i) $f(x) = x^4$

(ii) $f(x) = \cos(-3x)$

(iii) $f(x) = \sin\left(\dfrac{1}{2}x\right)$

(iv) $f(x) = -4x^{-\frac{1}{4}} - 3e^{-2x} - 3\ln(-2x)$ (where $x < 0$) [6]

7. Find the following integrals.

(a) $\displaystyle\int 1\,dx$ [1]

(b) $\displaystyle\int_{-1}^{1} x^4\,dx$ [2]

(c) $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin(2x)\,dx$ [2]

(d) $\displaystyle\int e^{-2x} - x^{-\frac{4}{5}}\,dx$ [2]

8. (a) For the list of numbers $0, 3, 3, -6, 4, 6, 0, 2, -3$, find the

(i) Mean

(ii) Median

(iii) Mode(s)

(iv) Interquartile range [5]

(b) Find the line of best fit using the least squares method with the points $(-4, 3), (-2, 1), (0, 1), (3, -2)$ and $(5, -5)$. [7]

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